Supersymmetric Seesaw Inflation

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ABSTRACT

Supersymmetric Unified theories which incorporate a renormalizable Type I seesaw mechanism for small neutrino masses can also provide slow roll inflection point inflation along a flat direction associated with a gauge invariant combination of the Higgs, slepton and right handed sneutrino superfields. Inflationary parameters are related to the Majorana and Dirac couplings responsible for neutrino masses with the scale of inflation set by a right-handed neutrino mass $M_{\nu^c} \sim 10^6 - 10^{12}$ GeV. Tuning of the neutrino Dirac and Majorana superpotential couplings and soft Susy breaking parameters is required to enforce flatness of the inflationary potential. In contrast to previous inflection point inflation models the cubic term is dominantly derived from superpotential couplings rather than soft A-terms. Thus since $M_{\nu^c} >> M_{Susy}$ the tuning condition is almost independent of the soft supersymmetry breaking parameters and therefore more stable. The required fine tuning is also less stringent than for Minimal SUSY Standard Model (MSSM) inflation or Dirac neutrino "A-term" inflation scenarios due to the much larger value of the inflaton mass. Reheating proceeds via 'instant preheating' which rapidly dumps all the inflaton energy into a MSSM mode radiation bath giving a high reheat temperature $T_{rh} \approx M_{\nu^c}^{\frac{3}{4}} 10^6$ GeV $\sim 10^{11} - 10^{15}$ GeV. Thus our scenario requires large gravitino mass > 50 TeV to avoid a gravitino problem. The 'instant preheating' and Higgs component of the inflaton also imply a 'non-thermal' contribution to Leptogenesis due to facilitated production of right handed neutrinos during inflaton decay. We derive the tuning conditions for the scenario to work in the realistic New Minimal Supersymmetric SO(10) GUT and show that they can be satisfied by realistic fits.

1

1 Introduction

Primordial inflation is perhaps the simplest dynamical mechanism which can explain the seed fluctuations[1] for the cosmic microwave background (CMB) radiation, and thus for the formation of large scale structures. Although a large number of inflationary models exist in the literature the majority of them are not grounded in any realistic model of particle physics, thus leaving them unconstrained by anything beyond the few parameters so far gleaned from measurements [2, 3] of the Cosmic Microwave background. Models where inflation is driven not by a generic scalar field but by an inflaton intimately tied[4] to the Standard Model gauge group and spectrum carry an obvious appeal. Moreover in order to have a successful (and calculable) reheating into the Standard Model degrees of freedom as is required for the success of Big Bang Nucleosynthesis, the inflaton must carry definite Standard Model gauge and Yukawa charges so that the inflaton condensate can efficiently decay into SM degrees of freedom after the end of inflation.

The suggestion [4] that inflation can be naturally embedded within the Minimal supersymmetric (SUSY) Standard Model (MSSM), with generic gravity mediated (i.e $N = 1$ supergravity type : but we assume canonical Kähler potential) soft supersymmetry breaking terms, is an attractive scenario which enables us to connect the microscopic origin of inflation to cosmological evolution on the largest scales. Models of this type are typically based on slow roll inflation associated with "flat directions" in the MSSM field space (along which the D-term potential vanishes). A well known theorem [5] allows one to use holomorphic gauge invariants formed from chiral superfields as coordinates for the D-flat manifold of the scalar field space of SUSY gauge theories. The flat directions are lifted by supergravity generated soft supersymmetry breaking terms and by non renormalizable terms in the MSSM effective superpotential. Such models (also called "A-Term Inflation" models [6, 7, 8]) typically require a fine tuning between the soft terms to ensure an inflection or saddle point of the field potential where the vacuum energy density drives a burst of inflation but nevertheless allows "graceful exit" due to the absence of a local minimum and the associated potential barrier which would prevent exit. In such models the (usually non-renormalizable) terms that lift D-flatness of the inflaton potential are hypothesized rather than deduced from a well defined underlying renormalizable model. Thus while they answer some of the relevant issues they have much scope for improvement. One may consider how to deduce the effective non-renormalizable superpotential by integrating out heavy fields from an underlying theory, or one may look for minimal extensions of the MSSM which may (like inflationary GUTs) support inflation even at the renormalizable level.

The first definite signal of physics beyond the SM came from neutrino oscillations which are now accepted as evidence of non zero neutrino masses in the milli-eV range. However the nature of neutrino masses, i.e whether they are of Dirac or Majorana type, is still unsettled. In the first case light neutrino masses are understood as being the consequence of highly suppressed Yukawa couplings, ($\mathcal{L} = y_\nu \overline{N}_R H_u L + ...; y_\nu \sim m_\nu / M_W \sim 10^{-12}$) 7 or more orders of magnitude smaller than the charged fermion Yukawa couplings. To be

dominantly of Dirac type these masses should be accompanied by highly suppressed right handed Majorana neutrino masses, $M_{\nu^c} \sim 0.1 eV$ or less. Conversely one may generate small effective (Type I seesaw[9]) neutrino Majorana masses $(m_\nu \sim (m_\nu^D)^2/M_{\nu^c})$ for the left handed neutrinos if the right handed neutrino masses M_{ν^c} take the large values permitted by their vanishing SM gauge charges. In this case the Dirac masses of the neutrinos need not be suppressed by ultra small Yukawa couplings as required in the Dirac mass case.

In [6, 7] an intriguing connection was made between the smallness of the (Dirac) neutrino masses and flatness of the inflaton potential within the MSSM extended by the addition of $U(1)_{B-L}$ gauge group and right handed neutrinos. The inflaton field was a gauge invariant D-flat direction, NH_uL, where N is the right handed sneutrino, H_u is the MSSM Higgs which gives masses to the up-type quarks, and L is the slepton field. The gauge invariant superpotential term $y_\nu N H_u L$ generates the tiny (Dirac) neutrino masses due to the aforementioned tiny neutrino Yukawa coupling ($y_\nu \sim 10^{-12}$). When coupled with soft trilinear and bilinear supersymmetry breaking terms of mass scale $\sim 100\ GeV$ to $10\ TeV$ the associated *renormalizable* inflaton potential can then be fine tuned to achieve inflection point inflation consistent with Wilkinson Microwave Anisotropy Probe(WMAP) 7 year data[6, 7].

Since the seesaw[9] explanation for neutrino masses is arguably preferable to the ad-hoc small Dirac masses explanation it is natural to ask if it too supports inflation. *Prima facie* such a scenario could face obstacles in meeting the requirements of the neutrino-inflaton scenario i.e ultra small superpotential couplings, and TeV scale trilinear/mass terms. Generic Type I seesaw relies upon large right handed neutrino Majorana masses which are generated by breaking of $B-L$ symmetry by vevs $V_{B-L} >> 10^{10} GeV$. An inflaton involving the right handed sneutrino will then have (supersymmetric) mass contributions as large as the righthanded neutrino mass. The cogency of the seesaw lies in not artificially singling out neutrino Yukawas to be ultra small. With large V_{B-L} the Dirac coupling of the neutrino need not be suppressed by hand. Indeed for, normal hierarchy, one obtains the third generation light neutrino masses $m_\nu \sim y_3^2 v_{EW}^2/M_{\nu^c} \sim 0.1 eV$ for $y_3 \sim 1, M_{\nu^c} \sim 10^{15}$ GeV. Such large couplings and masses would completely destroy the needed flatness of the inflationary potential. However reflection shows that a negative conclusion may be unwarranted since at least three generations of neutrinos and their superpartners are in play. So there is considerable scope for much smaller superpotential couplings: the neutrino Yukawa coupling eigenvalues could have the typical values associated with up type fermions while off diagonal components matched the tiny Majorana couplings in smallness. Off diagonal flat directions ($N_A H L_B, A \neq B$=1,2,3) can serve just as well as diagonal ones , in fact we shall see they are required in the realistic New Minimal SO(10) GUT implementation of our scenario .

Furthermore the popular Leptogenesis[10] scenario strongly hints at right handed neutrino masses in the range 10^6 to 10^{12} GeV. So for $V_{B-L} \sim M_X > 10^{16}$ GeV the superpotential couplings $f_A, A = 1, 2, 3$ (we will work in a basis where these couplings are diagonal), which generate right handed neutrino masses $M_{\nu_A^c} \sim f_A V_{B-L}$, are very small ($f_A \sim 10^{-9}$ to 10^{-4}). Thus the required ingredients for an inflaton in the Type I seesaw scenario are already present. Note that since generic Type I seesaw requires that $B - L$ is broken at

a high scale, issues concerning the efficient decay of the conjugate sneutrino component of the inflaton via their Yukawa couplings will need to be addressed. Reheating in our scenario proceeds via the so called 'instant pre-heating' mechanism[11] resulting in a high reheat temperature due to rapid dump of the inflaton energy into MSSM modes. The Higgs component of the inflaton implies[12] a non-thermal contribution to leptogenesis.

Issues regarding natural values for superpotential couplings come into focus when viewed in the context of the so called Minimal Left Right supersymmetric models[13] and their embedding in GUT models[14, 15]. SUSY Left-Right Models are advantaged due to their protection of R-parity as a gauged discrete symmetry, which provides a stable lightest supersymmetric particle (LSP). They simultaneously and naturally implement Seesaw mechanisms for neutrino masses[13]. Moreover such models have also been incorporated in the realistic and predictive New Minimal Susy SO(10) grand unified theories(NMSGUT)[16, 17] where all the hard parameters of the MSSM are fitted in terms of fundamental parameters of the GUT and soft SUSY breaking parameters (of the Non-Universal Higgs masses (NUHM) type) defined at the Unification scale $M_X \sim 10^{16} - 10^{18}$ GeV. Such GUTs have viable Bino dark matter candidates and make distinctive predictions for the type of SUSY spectra observable at the LHC. In 2008, well before the discovery of Higgs mass of around 125 GeV in 2011-2012 and the consequent realization that a general framework such as the phenomenological MSSM (pMSSM) requires that the soft trilinear couplings $A_{t,b}$ be *large*, we concluded[16] that *the NMSGUT would be falsified* by its failure to fit the down type quark masses *unless* A_0, μ were in the 10's of TeV : leading to a mostly decoupled superspectrum with only the LSP, gauginos and possibly a light slepton in the sub-TeV range ! The experimental data has now forced this realization on practitioners of MSSM parametrology[18]. In the NMSGUT it was a pre-diction. In the NMSGUT the successful fitting of fermion masses necessarily entails ultrasmall neutrino Majorana-Yukawa couplings leading to (first generation) right handed neutrino Majorana masses as small as 10^6 GeV. Taken together with the possibility of small values for the light generation Yukawa Dirac couplings it is possible to implement viable inflection point inflation by suitable tuning at the supersymmetric level itself. This is technically more appealing than a tuning applied to soft susy parameters which, being unprotected by SUSY, are unstable. We derive the tuning conditions for the NMSO(10)GUT and show how to satisfy them explicitly.

In Section **2** we review and summarize the generic renormalizable single scalar inflaton inflection point model and calculate its slow-roll parameters, power spectrum and spectral index so as to use these results with the supersymmetric Type I Seesaw model(SIMSSM), with supergravity soft terms, once we have shown that it generates a suitable potential of the renormalizable type. In Section **3** we see how a generic Supersymmetric $SU(3) \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ model, with generic supergravity type soft supersymmetry breaking terms, which contains the essentials of the Type I Susy seesaw implemented in the Minimal Susy LR Models and in R-parity preserving Susy GUTs, provides an attractive Inflationary scenario in parallel with its achievement of realistic neutrino masses. In Section **4** we discuss the general features of reheating in this model and remark on the types of

Leptogenesis that can arise from the inflaton and right handed neutrino decay. In Section **5** we give a discussion of the embedding in the NMSGUT and the inflationary parameters associated with realistic fits. We conclude with a brief discussion.

2 Generic Renormalizable Inflection point inflation

A generic renormalizable inflection point inflation model can be formulated in terms of a single complex field φ. Such a model[6, 7] can reproduce the observed[3] inflationary power spectrum $P_R = (2.43 \pm 0.11) \times 10^{-9}$, spectral index $n_s = .967 \pm 0.014$ and scale invariance $k dn_s/dk \simeq 0$. It is also relevant to note that the ratio of tensor to scalar power spectrum amplitudes $r = \frac{P_T}{P_R}$ is known to be less than about 0.5. After extremizing with respect to the angular degree of freedom (which has positive curvature and cannot support inflation) one is left with the potential for a real degree of freedom ϕ in the complex scalar inflaton field φ

$$V = \frac{h^2}{12}\phi^4 - \frac{Ah}{6\sqrt{3}}\phi^3 + \frac{M^2}{2}\phi^2 \tag{2.1}$$

Here A, h, ϕ are real and positive without loss of generality. The formulae we derive in this section are applicable to any single inflaton theory with a renormalizable potential.

In the model of [6] the A, M receive dominant contributions from trilinear and quadratic soft supersymmetry breaking parameters: $A, M \sim 10^2 - 10^4$ GeV. A very small neutrino Yukawa coupling $\sim 10^{-12}$ and a high degree of fine tuning between A and M is necessary to reproduce the observed inflation parameters[7, 8]. In our work however the contributions from soft supersymmetry breaking terms play a negligible role. The controlling mass scale is much higher , the required size of the yukawa couplings is larger and the degree of fine tuning is much less.

It is convenient to trade the parameter A for a fine-tuning parameter Δ by replacing $A = 4M\sqrt{1-\Delta}$ ($\Delta = \beta^2/4$ in the notation of [7]). The inflection point at

$$\phi_0 = \frac{\sqrt{3}M}{h}(1 - \Delta + O(\Delta^2)) \qquad : \qquad V''(\phi_0) = 0 \tag{2.2}$$

is also a saddle point ($V'(\phi_0) = 0$) when $\Delta = 0$. For small Δ

$$\begin{aligned}
V(\phi_0) &= V_0 = \frac{M^4}{4h^2}(1 + 4\Delta) \qquad ; \qquad V'(\phi_0) = \alpha = \frac{\sqrt{3}M^3\Delta}{h} \\
V'''(\phi_0) &= \gamma = \frac{2Mh}{\sqrt{3}}(1 - 2\Delta)
\end{aligned} \tag{2.3}$$

If the coupling h is tiny $V_0 >> M^4$ and $\phi_0 >> M/h$. Notice that γ tends to be quite small due to the smallness of h, while α is small (but non-zero[8]) because it is tuned to be small. The large vacuum energy and flatness of the potential around ϕ_0 then imply that if ϕ starts with a value close to $\phi = \phi_0$ and a small field velocity the universe will execute slow roll inflation as the field ϕ rolls slowly down through a narrow field interval of width

$\Delta\phi \sim V_0/\gamma M_p^2$ below ϕ_0. Around the inflection point ϕ_0, we can write the inflection point inflation potential in the form

$$V(\phi) = V_0 + \alpha(\phi - \phi_0) + \frac{\gamma}{6}(\phi - \phi_0)^3 + \frac{h^2}{12}(\phi - \phi_0)^4 \tag{2.4}$$

The last term is essentially negligible since h^2 is very small by assumption.

The slow roll parameters are defined as($M_p = 2.43 \times 10^{18}\, GeV$)

$$\begin{aligned}
\eta(\phi) &= \frac{M_p^2 V''}{V} \simeq \frac{M_p^2}{V_0}\gamma(\phi - \phi_0) \\
\epsilon(\phi) &= \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \simeq \left(\alpha + \frac{\gamma}{2}(\phi - \phi_0)^2\right)^2\left(\frac{M_p^2}{2V_0^2}\right) \\
\xi &= \frac{M_p^4 V' V'''}{V^2} \simeq \frac{M_p^4 \alpha\gamma}{V_0^2}
\end{aligned} \tag{2.5}$$

The small first and third Taylor coefficients α, γ determine[8, 20, 19] the measured parameters of inflation (P_R, n_s) once the field values (ϕ_{CMB}, ϕ_{end}) at the time of horizon entry of the "pivot" momentum scale ($k_{pivot} = 0.002$ Mpc^{-1}) and at termination of the slow roll are fixed[8, 19](on the basis of an overall cosmogonic scenario and the consistency of the slow roll approximation $(\eta(\phi_{end}) \approx 1)$ respectively). k_{pivot} corresponds to a representative scale of current cosmological observations. The field value at the beginning of inflation is of notional interest only. It is the number $(N_{CMB} = N(\phi_{CMB}))$ of e-folds of inflation left to occur after field value ϕ_{CMB} reached at the time when the fluctuation scale of interest(k_{pivot}) left the comoving horizon (i.e $k = a_k H_k$) during inflation that is of significance. This number is determined by the overall history of the Universe from primordial times[19]. Plausible inflationary cosmogonies require $40 < N_{CMB} < 60$ and this severely restricts the inflation exponents.

The field value at the end of slow roll inflation ϕ_{end} is defined as the value where

$$\eta(\phi_{end}) \simeq \frac{\gamma(\phi_{end} - \phi_0)M_p^2}{V(\phi_0)} \simeq 1 \tag{2.6}$$

which gives

$$\phi_0 - \phi_{end} = \frac{V_0}{\gamma M_p^2}, \tag{2.7}$$

Then in the slow roll approx $\ddot{\phi} \simeq 0, \dot{\phi} = -V'(\phi)/3H$, where $H = \sqrt{V(\phi_0)/(3M_p^2)}$ is the (constant) inflation rate during slow roll inflation, one has

$$\begin{aligned}
N(\phi) &= -3\int_\phi^{\phi_{end}} \frac{H^2}{V'(\phi)}d\phi \\
&= \sqrt{\frac{2}{\alpha\gamma}}\frac{V_0}{M_p^2}\left(\arctan\sqrt{\frac{\gamma}{2\alpha}}(\phi_0 - \phi_{end}) - \arctan\sqrt{\frac{\gamma}{2\alpha}}(\phi_0 - \phi)\right) \tag{2.8}
\end{aligned}$$

and conversely

$$\phi(N) = \frac{\phi_{end} + \phi_0(\phi_0 - \phi_{end})\sqrt{\frac{\gamma}{2\alpha}}\tan\sqrt{\frac{\alpha\gamma}{2}\frac{NM_p^2}{V_0}} + \sqrt{\frac{2\alpha}{\gamma}}\tan\sqrt{\frac{\alpha\gamma}{2}\frac{NM_p^2}{V_0}}}{1 + (\phi_0 - \phi_{end})\sqrt{\frac{\gamma}{2\alpha}}\tan\sqrt{\frac{\alpha\gamma}{2}\frac{NM_p^2}{V_0}}} \quad (2.9)$$

It is worth remarking that this inversion of the function $N(\phi)$ was derived without assuming that $\phi_{end} << \phi(N)$ [8]. Together with an interpolating function derived below it allows us to obtain analytic formulae for the relations required among the parameters of the inflationary potential for successful inflation: avoiding tedious and opaque graphical methods [7].

The observed Cosmic Microwave Background(CMB) data [3] pose constraints on the power spectrum and spectral index for modes around the pivot scale. Barring non-standard scenarios where the post-inflationary period is punctuated by episodes of modified expansion, the number of remaining e-folds at the time the pivot scale left the horizon during inflation may be estimated by using the standard Big Bang thermal cosmogony along with estimates of the reheating behaviour of the universe after inflation. This gives[19]

$$N_{\text{pivot}} = 65.5 + \ln\frac{\rho_{rh}^{\frac{1}{12}}V_0^{\frac{1}{6}}}{M_{\text{p}}} \quad (2.10)$$

where ρ_{rh} is the energy density after reheating and V_0 the potential value during inflation. The reheating behaviour of the NLH flat direction inflaton is quite different from the the original quadratic chaotic inflation models for conjugate sneutrino inflation[21, 22]. The tripartite composition of the inflaton out of $L, H, \tilde{\nu}^c$ degrees of freedom will ensure that the bulk of the energy in the inflaton will be dumped into light degrees of freedom on the very first oscillation. For the present we merely assume that the reheating is immediate so that one can set $\rho_{rh} = V_0$ in N_{Pivot}. We then find that for M in the range $10^6 - 10^{12}\,GeV$, h lies between $10^{-9.5}$ to $10^{-6.5}$ and then $N_{pivot} = N_{CMB} = 51 \pm 5$ adequately covers the possible range. Even if this range is lowered by effects of reheating or non-standard cosmogonies the effect on the relevant exponents will prove to be marginal. Although the observed CMB is actually a combined spectrum of modes exiting the horizon around $|N - N_{\text{pivot}}| \leq 5$, we can approximate and regard it as the single spectrum from the mode that exits the co-moving horizon when $\phi = \phi_{CMB}$ only. Thus $\phi = \phi_{CMB}$ is the field value near ϕ_0 where the inflation giving rise to observable effects today kicks in (when N_{CMB} e-folds of inflation are remaining). The power spectrum and spectral index we see today are then $P_R(\phi(N_{\text{CMB}}))$ and $n_s(\phi(N_{\text{CMB}}))$ respectively, where $|N_{CMB} - N_{\text{pivot}}| < 5$.

The slow roll inflation formula for the power spectrum of the mode that is leaving horizon when the inflaton rolls to ϕ is([20])

$$P_R(\phi) = \frac{V_0}{24\pi^2 M_{\text{p}}^4 \epsilon(\phi)}, \quad (2.11)$$

and the corresponding spectral index and it's variation with momentum is

$$
\begin{aligned}
n_s(\phi) &\equiv 1 + 2\eta(\phi) - 6\epsilon(\phi) \\
\mathcal{D}_k(n_s) &= \frac{k dn_s(\phi)}{dk} = -16\epsilon\eta + 24\epsilon^2 + 2\xi^2
\end{aligned}
\tag{2.12}
$$

The ratio of tensor to scalar perturbations $r = \frac{P_T}{P_R} = 16\epsilon$. In practice ϵ, ξ are so small in the narrow region near ϕ_0 where slow-roll inflation occurs that their contribution to n_s is negligible. Thus $\mathcal{D}_k(n_s)$ is negligible i.e. the spectral index is scale invariant in the observed range, as is allowed by observation so far.

To search for sets of potential parameters M, h, Δ compatible with P_R, n_S, N_{CMB} in their allowed ranges one may proceed as follows. First one uses the chosen (within experimental range) values of P_R, n_s and given M, h to define

$$
\begin{aligned}
\epsilon_{CMB} &= \frac{V_0}{24\pi^2 M_p^4 P_R} \\
\eta_{CMB} &= \frac{(n_s - 1)}{2}
\end{aligned}
\tag{2.13}
$$

From $\epsilon_{CMB}, \eta_{CMB}$ one may deduce α_{CMB}, ϕ_{CMB} using the eqns.(2.5)

$$
\begin{aligned}
\phi_{CMB} &= \phi_0 + \frac{V_0 \eta_{CMB}}{\gamma M_p^2} \\
\alpha_{CMB} &= \sqrt{2\epsilon_{CMB}} \frac{V_0}{M_p} - \frac{V_0^2 \eta_{CMB}^2}{2\gamma M_p^4}
\end{aligned}
\tag{2.14}
$$

The required fine-tuning Δ is then

$$
\Delta = \frac{h\alpha_{CMB}}{\sqrt{3}M^3} = (\frac{M}{4hM_p})^4 (\frac{16h^2 M_p}{3\pi M \sqrt{P_R}} - (1 - n_s)^2)
\tag{2.15}
$$

α_{CMB}, Δ should emerge real and positive and using $\{\alpha_{CMB}, \phi_{CMB}\}$ in the formula for N_{CMB} one should obtain a sensible value in the range $N_{CMB} = 51 \pm 5$. Positivity of Δ (a local minimum develops if Δ is negative leading to eternal inflation) requires

$$
h^2 \geq M \frac{3\pi(1 - n_s)^2 \sqrt{P_R}}{16 M_P}
\tag{2.16}
$$

Using eqns.(2.3,2.72.13) in eqn(2.8) we have

$$
\begin{aligned}
N_{CMB} &= \frac{1}{z} \arctan \frac{2z(1 + n_s)}{8z^2 + 1 - n_s}
\end{aligned}
\tag{2.17}
$$

$$
z = (\frac{h^2 M_p}{3\pi M \sqrt{P_R}} - \frac{(1 - n_s)^2}{16})^{\frac{1}{2}}
\tag{2.18}
$$

By solving eqn.(2.17) for $z = z_0(N_{CMB}, n_s)$ one obtains the general relation between h and M :

$$
\frac{h^2}{M} = \frac{3\pi\sqrt{P_R}}{M_P}(z_0^2(N_{CMB}, n_s) + \frac{(1 - n_s)^2}{16})
\tag{2.19}
$$

and then

$$\Delta = \frac{16 M^2 z_0^2(N_{CMB}, n_s)}{9\pi^2 M_P^2 P_R((1-n_s)^2 + 16 z_0^2(N_{CMB}, n_s))^2} \tag{2.20}$$

Where $z_0(N_{CMB}, n_s)$ is the solution of eqn (2.17). An excellent approximation to the the required function in the region of interest in the N_{CMB}, n_s plane is given by the Taylor series around $n_s^0 = 0.967, N_C^0 = 50.006$:

$$z_0(N_{CMB}, n_s) = .0238 - 0.0006(N_{CMB} - N_C^0) + 0.2407(n_s - n_s^0)$$
$$+0.000022\frac{(N_{CMB} - N_C^0)^2}{2} - 3.70875\frac{(n_s - n_s^0)^2}{2} + 0.002353(N_{CMB} - N_C^0)(n_s - n_s^0)$$
$$-.0000015\frac{(N_{CMB} - N_C^0)^3}{6} + 8.79982\frac{(n_s - n_s^0)^3}{6} - 0.000788\frac{(N_{CMB} - N_C^0)^2(n_s - n_s^0)}{2}$$
$$-0.7536\frac{(N_{CMB} - N_C^0)(n_s - n_s^0)^2}{2} \tag{2.21}$$

In Fig. 1 we have plotted the contours of $z_0(N_{CMB}, n_s)$ in the N_{CMB}, n_s plane and one sees that the variation of z_0 is rather modest. So for the plausible range $46 < N_{CNB} < 56$ one obtains a tight constraint on the exponents in the relation between h,Δ and M:

$$h^2 \sim 10^{-24.95\pm0.17}(\frac{M}{GeV}) \quad ; \quad \Delta \sim 10^{-28.17\pm.13}(\frac{M}{GeV})^2 \tag{2.22}$$

We have estimated the maximum variations in the exponents corresponding to the quoted errors in the WMAP 7- year data [3] from the graphs in Fig. 2 and Fig. 3.

However a clearer qualitative understanding results from noticing that for $N_{CMB} \sim 50$, $Z_0 \approx \frac{1.2}{N_{CMB}}$ solves eqn.(2.17) to a good approximation. Then eqn.(2.18) gives

$$\frac{h^2}{M} \approx \frac{3\pi}{M_P}\frac{\sqrt{P_R}}{N_{CMB}^2} \approx \frac{2.75 \times 10^{-22}}{N_{CMB}^2} \approx 10^{-25} \tag{2.23}$$

$$\frac{\Delta}{M^2} \approx \frac{4.14 \times 10^{-34}}{N_{CMB}^2 P_R} \approx 10^{-28.2} GeV^{-2} \tag{2.24}$$

Thus these simple approximate expressions give effectively the same results as the more carefully derived expressions in eqn.(2.22). Thus we have viable inflation with

$$V_0 \sim \frac{M^4}{h^2} \sim (M)^3 \times 10^{25}\, GeV \sim 10^{43} - 10^{61}\, GeV^4 \tag{2.25}$$

$$H_0 \sim \sqrt{\frac{V_0}{M_P^2}} \sim 10^3 - 10^{12}\, GeV \quad ; \quad T_{max} \sim V_0^{\frac{1}{4}} \sim 10^{11} - 10^{15}\, GeV \tag{2.26}$$

It is clear from eqn.(2.22) that the fine-tuning measure grows with M so that $\beta = \sqrt{\Delta}$ can be as large as 10^{-2} for $M \sim 10^{12}$ GeV. Due to the large value of the inflaton mass compared to the case of MSSM inflation[4] or Dirac neutrino inflation[6, 8] the fine-tuning of parameters required is much less severe and no additional dynamics need be invoked
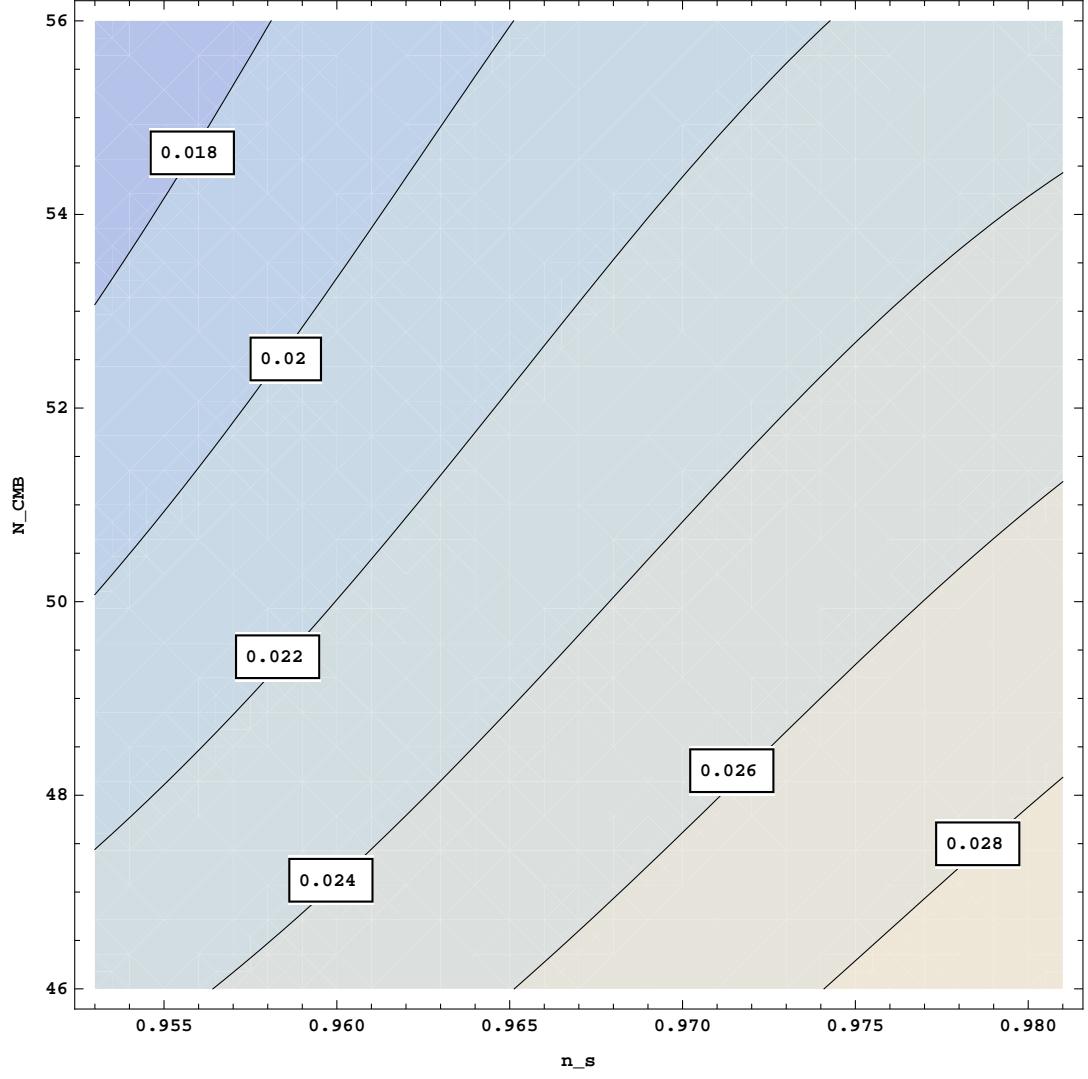
Figure 1: z_0 contours in the N_{CMB}, n_s plane. The variation shown contributes to the small range of permitted magnitudes for $h^2/M, \Delta/M^2$ etc
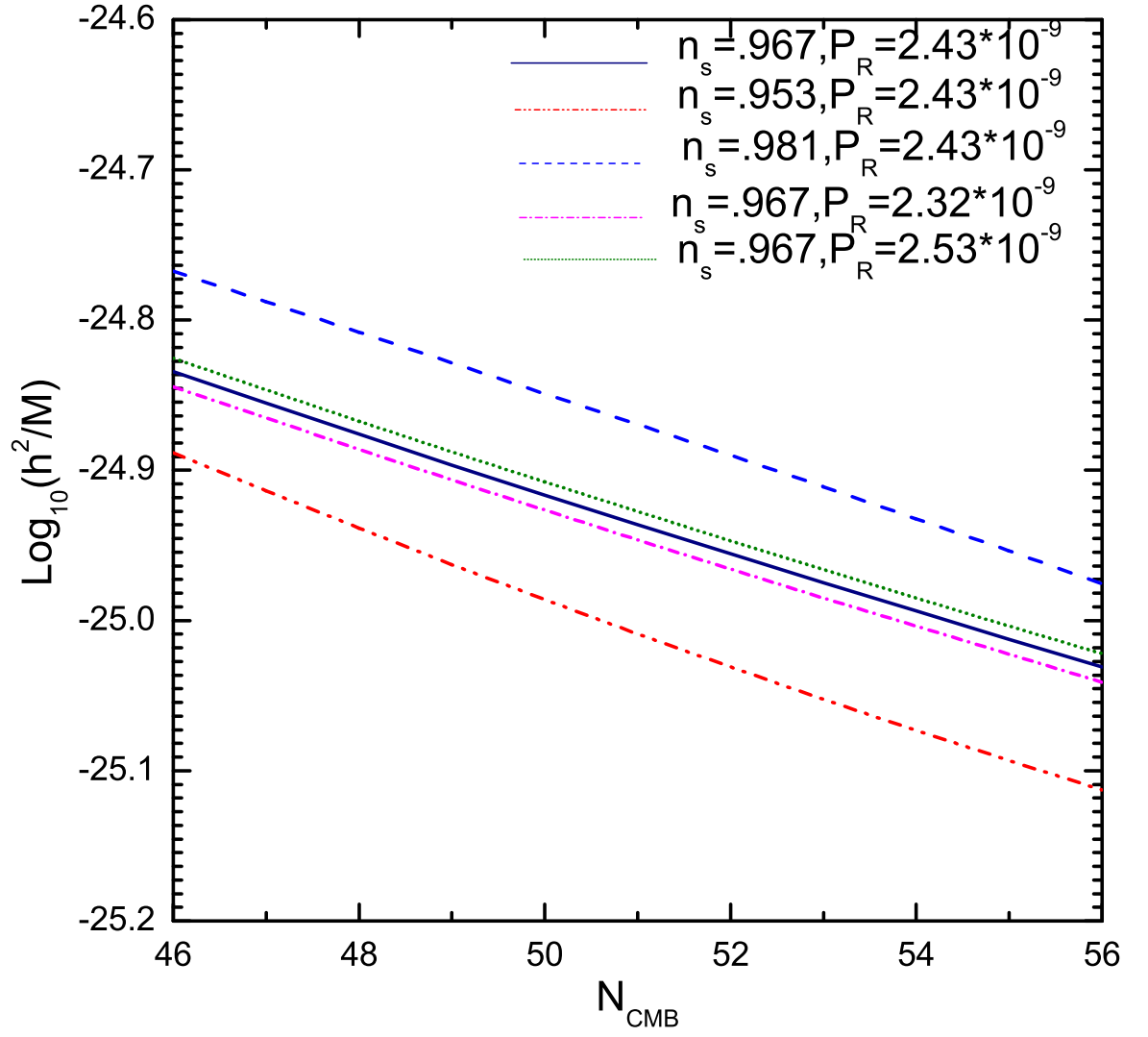
Figure 2: Variation of exponent of $\frac{h^2}{M}$ with N_{CMB} for different values of n_s, P_R.
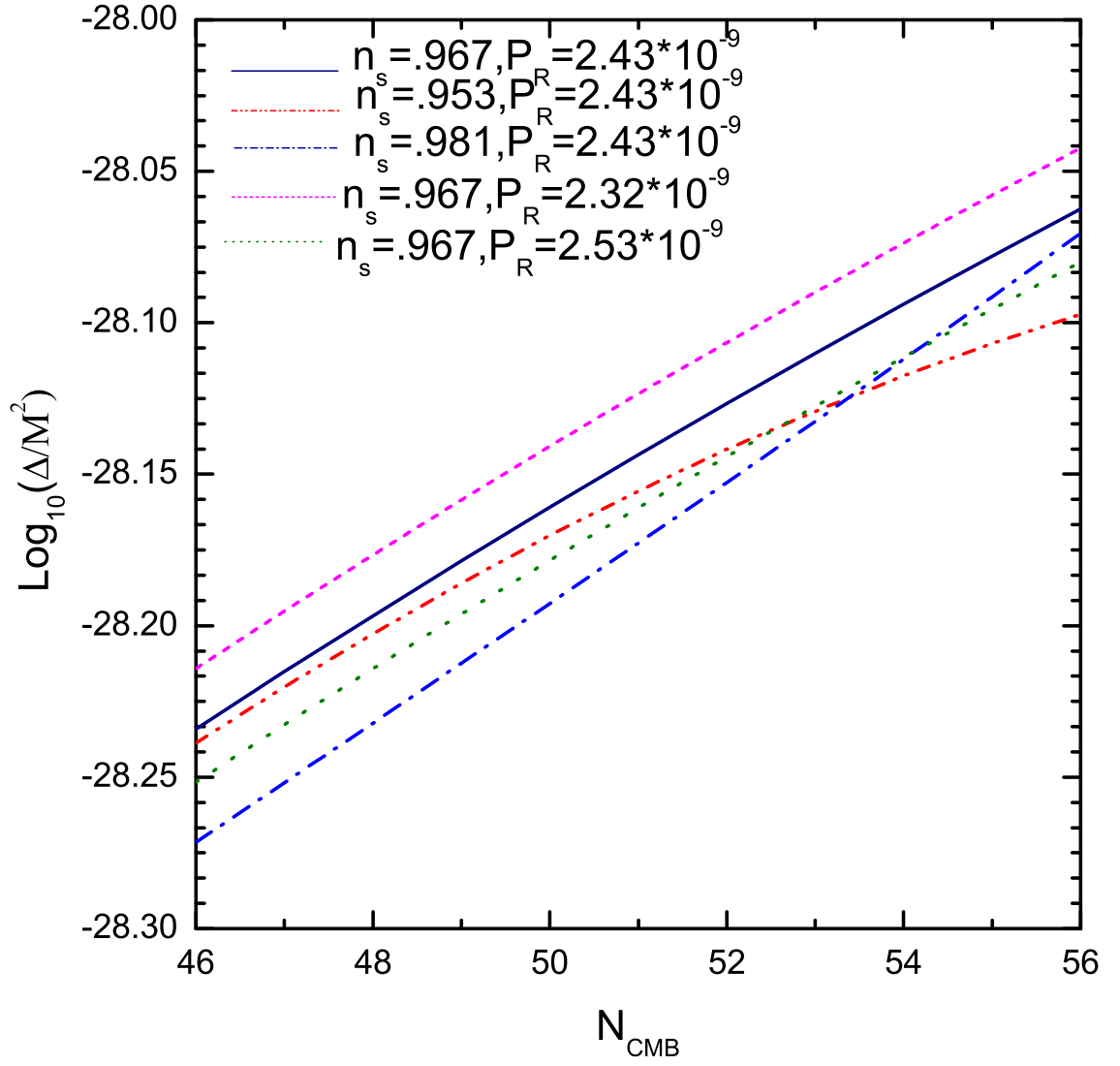
Figure 3: Variation of exponent of $\frac{\Delta}{M^2}$ with N_{CMB} for different values of n_s, P_R.

to make it plausible[7, 23]. It is also important to note that the ratio of tensor to scalar perturbations $r \simeq 16\epsilon \simeq 2(M/10^{14}GeV)^3$. Since M is at most the heaviest right handed neutrino mass $\sim 10^{12}$ GeV it is clear that it is difficult to get $r > 10^{-6}$. Thus measurement of tensor perturbations via the Cosmic Microwave background polarization at the $r \sim 10^{-3}$ level or larger would not be compatible with inflection point inflation controlled by the right handed neutrino mass. Any renormalizable single inflaton model must respect these generic constraints and yield values of its associated parameters that are sensible in terms of the other (particle) physics that it describes. It remains to specify the Renormalizable Susy seesaw Inflaton scenario and consider the NMSGUT as a self contained realistic test bed.

3   Supersymmetric seesaw Inflaton model

The essentials of the Supersymmetric seesaw inflation scenario may be captured by considering a model with gauge group $SU(3) \times SU(2) \times U(1)_R \times U(1)_{B-L}$ and the field content of the MSSM with some additional superfields. Soft supersymmetry breaking terms are of the supergravity type [i.e trilinears proportional to yukawa couplings and universal, or universal except for Higgs (NUHM scenario), soft scalar masses]. The essential fields beyond the MSSM consist of a right handed Neutrino chiral multiplet $N[1, 1, -1/2, 1]$ and a field $S[1, 1, 1, -2]$ whose vev generates the large Majorana masses M_ν ($10^6 - 10^{14}$ GeV) for the conjugate neutrinos $\nu_A^c \equiv N_A$ via a renormalizable superpotential coupling $3\sqrt{2}f_{AB}S\nu_A^c\nu_B^c$. Additional fields Θ_i which serve to fix the vev of S are also present as in Minimal Supersymmetric Left Right Models (MSLRMs)[13] and in GUTs that embed them [14, 15, 16]. The other essential component of the scenario is neutrino Dirac mass generating Yukawa couplings $y_{AB}, A, B = 1, 2, 3$ in the superpotential. These couple the right handed neutrinos to the Left chiral lepton doublets $L_A = \begin{pmatrix} \nu & e \end{pmatrix}_A^T, A = 1, 2, 3$. L_A transform as $L[1, 2, 0, -1]$ and the up type Higgs doublet type field as $H[1, 2, 1/2, 0]$ so that $y_{AB}NL_AH_B$ is a gauge invariant term in the Superpotential. Of course each such doublet present in the underlying theory must have its complementary doublet transforming as e.g. $\overline{H}[1,2,-1/2,0]$ to cancel anomalies. The relevant flat direction is assumed to extend out of the minimum of the supersymmetric potential corresponding to the breaking of the gauge group down to the MSSM symmetry

$$SU(3) \times SU(2) \times U(1)_R \times U(1)_{B-L} \rightarrow SU(3) \times SU(2) \times U(1)_Y \qquad (3.1)$$

This leads to a Type I seesaw plus MSSM (SIMSSM) effective theory.

After the breaking one has $Y = 2T_{3R} + (B - L)$ where T_{3R} is the $U(1)_R$ generator. Note that unlike the case of the Dirac neutrino masses scenario [6] $B-L$ is *not* a gauge symmetry down to low energies. This can have important consequences for nucleosynthesis and matter domination since the heavy right handed neutrinos must find a non-gauge channel to decay through. In the present case this channel must perforce be a Yukawa coupling since the right-handed neutrinos are singlets of the low energy (SM) gauge group. This is in contrast to the Dirac scenario where a low scale of B-L breaking is assumed so that ν^c can decay via gauge couplings.

The fields $\tilde{N}$(the chosen conjugate sneutrino), $\tilde{\nu}$(chosen left sneutrino flavour from a Lepton doublet L with suitable Yukawa couplings) and the light neutral Higgs field (from the doublet H with $Y = +1$) may be parametrized in terms of the flat-direction associated with the gauge invariant NLH as

$$\tilde{N} = \tilde{\nu} = h_0 = \frac{\varphi}{\sqrt{3}} = \phi e^{i\theta}; \quad \phi \geq 0, \quad \theta \in [0, 2\pi) \tag{3.2}$$

The additional fields Ω_i, unspecified at the moment, are assumed to be coupled to S in such a way that extremization of the SUSY potential using $F_{\Omega_i} = 0, \quad D_\alpha|_{\phi=0} = 0$ fixes the vev of S: $< S >= \bar{\sigma}/\sqrt{2}$ without constraining the inflaton field φ. This is of course true in the Minimal Susy LR models[13] and renormalizable Susy SO(10) GUTs [15, 16] which are our inspiration.

The vanishing of the D-term for the $B-L$ generator requires Ω_i to include the companion field(s) $\overline{S}[1, 1, -1, 2]$ which have a vev of equal magnitude as S in order to preserve SUSY through the symmetry breaking down to the MSSM symmetry at high scales. This is just as in MSLRMs and R-parity preserving GUTs [13, 14, 15, 16]. The gauge invariance of NLH ensures that the D-terms for the flat direction vanish. Thus at scales $\phi \sim \bar{\sigma} >> M_S$ where SUSY is exact the relevant superpotential is given by:

$$W = 3\sqrt{3}yN\nu h + 3f\sqrt{2}SNN + ... = y\varphi^3 + f\sqrt{2}S\varphi^2 + ... \tag{3.3}$$

where $h, f, \bar{\sigma}$ can be taken real without loss of generality. The right handed neutrino Majorana mass will be $M_\nu = 6f\bar{\sigma}$.

Since the equations of motion of the unperturbed vacuum imply $< F_S >= 0$, $< S >= \bar{\sigma}/\sqrt{2}$ this superpotential leads to a flat direction potential

$$
\begin{aligned}
V_{susy} &= |3y\varphi^2 + 2f\bar{\sigma}\varphi|^2 + 2|f\varphi^2|^2 \\
&= f^2 \left[(2 + 9\tilde{y}^2)\phi^4 + 12\tilde{y}\phi^3\bar{\sigma}\cos\theta + 4\bar{\sigma}^2\phi^2 \right]
\end{aligned} \tag{3.4}
$$

Here $\tilde{y} = y/f$ and we see that $\bar{\sigma}$ sets the mass scale. Minimizing with respect to θ gives $\theta = \pi$. In so far as we are here interested only in the inflationary dynamics (once parameters have been tuned to ensure an inflection point in the plateau region where $|\varphi| \sim \bar{\sigma}$) we can focus on just the real part of φ and set $\varphi = -\phi$ with ϕ real and positive near the inflection point but free to fall into the well around $\phi = 0$ and oscillate around that value. The imaginary part ϕ' of φ has a large curvature $V_{\phi'\phi'} \sim \bar{\sigma}^2$ in the plateau region. Since $V_{\phi'}|_{\phi'=0} = 0$ it is consistent to consider the dynamics in the real φ plane alone as a leading approximation. The effect of jitter in the ϕ' direction when the dynamics is initiated with $\phi' \neq 0$ can be studied numerically as a correction to the dynamics of the inflaton field ϕ.

In addition one also expects a contribution to the potential from the μ term for the Higgs doublets together with SUSY breaking quadratic and cubic soft terms, which we assume to be of the type generated by supergravity, but with non universal Higgs masses,

i.e of the form:

$$
\begin{aligned}
V_{soft} &= \left[A_0(y\varphi^3 + f\sqrt{2}S\varphi^2) + h.c \right] + m_{\tilde{f}}^2 \sum_{\tilde{f}} |\tilde{f}|^2 + m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 \\
&= f^2 \left[\tilde{y}\tilde{A}_0\phi^3\bar{\sigma}\cos 3\theta + \tilde{A}_0\bar{\sigma}^2\phi^2\cos 2\theta + \tilde{m}_0^2\bar{\sigma}^2\phi^2 \right]
\end{aligned}
\tag{3.5}
$$

here $\tilde{m}_0 = m_0/f\bar{\sigma}$, $\tilde{A}_0 = 2A_0/f\bar{\sigma}$. The soft mass m_0 receives contributions from the sfermion and Higgs soft masses as well as the μ term

$$
m_0^2 = \frac{2m_{\tilde{f}}^2 + \overline{m}_H^2}{3}
\tag{3.6}
$$

$m_{\tilde{f},H}$ are the sfermion and up type Higgs soft effective masses at the unification scale ($\overline{m}_H^2 = m_H^2 + |\mu|^2$). Since these masses and A_0 should be in the range $10^2 - 10^5$ GeV while the righthanded neutrino masses lie in the range $10^6 - 10^{12}$ GeV, it is clear that $\tilde{m}_0, \tilde{A}_0$ are small parameters and even for the large values of $m_0, A_0 \sim 10^5$ GeV found in the NMSGUT $\tilde{m}_0, \tilde{A}_0 << 1$. Thus these terms cannot significantly change $\theta = \pi$ assumed earlier. The total inflaton potential is then

$$
V_{tot} = f^2 \left((2 + 9\tilde{y}^2)\phi^4 - (\tilde{A}_0 + 12)\tilde{y}\bar{\sigma}\phi^3 + (\tilde{A}_0 + \tilde{m}_0^2 + 4)\bar{\sigma}^2\phi^2 \right).
\tag{3.7}
$$

Thus we have a generic quartic inflaton potential of the same type as in Section **2** but the parameter values in the case of Type I seesaw are quite different from the light Dirac neutrino case. We have the identification of parameters

$$
\begin{aligned}
h &= f\sqrt{12(2 + 9\tilde{y}^2)} \\
A &= \frac{3f(\tilde{A}_0 + 12)\tilde{y}\bar{\sigma}}{\sqrt{(2 + 9\tilde{y}^2)}} \\
M^2 &= 2f^2\bar{\sigma}^2(4 + \tilde{A}_0 + \tilde{m}_0^2) \\
\Delta &= (1 - \frac{A^2}{16M^2}) \\
&= \left(1 - \frac{9\tilde{y}^2(\tilde{A}_0 + 12)^2}{32(2 + 9\tilde{y}^2)(\tilde{A}_0 + \tilde{m}_0^2 + 4)} \right)
\end{aligned}
\tag{3.8}
$$

For seesaw models the natural magnitude for the neutrino Dirac mass is, $m_\nu^D > 1 MeV$ (i.e $|y_\nu^D| > 10^{-5}$ and then the limit $m_\nu << 0.01 eV$ for the lightest neutrino (assuming direct hierarchy) implies $M_{\nu^c} > 10^6$ GeV). Since the preferred values for the Susy breaking scale are smaller than 100 TeV (at most) it follows that the maximum value of $|\tilde{A}_0|, |\tilde{m}_0| \sim 0.1$ and they could be much smaller for more typical larger values of the conjugate neutrino masses $M_{\nu^c} \sim 10^8$ to 10^{12} GeV. It is then clear from the corresponding range $\Delta \sim 10^{-12}$ to 10^{-4} that the coupling ratio $\tilde{y} = y/f$ becomes ever closer to exactly $\tilde{y} = 4/3$ as M increases and even for $M \sim 10^6$ GeV differs from 1.333 only at the second decimal place. Thus to a

good approximation $h = 6\sqrt{6}f$. Then it follows from the Eqs.(2.22)and (3.8) that

$$
\begin{aligned}
f &\simeq 10^{-26.83\pm0.17}(\frac{\overline{\sigma}}{GeV}) \qquad ; \qquad M \simeq 10^{-25.38\pm0.17}(\frac{\overline{\sigma}}{GeV})^2 \\
\Delta &\simeq 10^{-78.93\pm0.47}(\frac{\overline{\sigma}}{GeV})^4
\end{aligned}
\tag{3.9}
$$

The range $M \sim 10^{6.6}$ to $10^{10.6}$ GeV corresponds nicely to $10^{16}\ GeV < \bar\sigma < 10^{18}\ GeV$: as is natural in single scale Susy SO(10) GUTs[14, 15, 16, 17]. f increases with $\bar\sigma$ with values below to 10^{-11} achievable in the NMSGUT only with difficulty. Of course in MSLRMs, since there are no GUT constraints on $\bar\sigma$, one can assume somewhat wider ranges for these parameters.

In all relevant cases $\Delta < 10^{-4}$ is required. Thus the above equations imply that $\tilde{y}^2$ must be close to the value

$$
\tilde{y}_0^2 = \frac{64}{9}\frac{4 + \tilde{A}_0 + \tilde{m}_0^2}{16 - 8\tilde{A}_0 - 32\tilde{m}_0^2 + \tilde{A}_0^2}
\tag{3.10}
$$

Here $\tilde{A}_0, \tilde{m}_0 \sim O(M_S/M_{\nu^c}) << 1$, hence $\tilde{y}_0$ is rather close to 4/3 and the equality is very close for larger $M \sim f\bar\sigma$ since then $\tilde{A}_0, \tilde{m}_0$ are tiny. This then is the type of fine tuning that supports the development of inflation in SIMSSM models. We see that the measure of severity of fine tuning $\beta = \sqrt{\Delta} \sim 10^{-2} - 10^{-6}$ compares quite favourably with the case of the MSSM or Dirac neutrino inflaton since there $\beta \sim 10^{-12}$ to 10^{-10} due to the low values of the inflaton mass in those cases. The dominant component of the fine tuning in the present case is a fine-tuning of superpotential parameters, which is radiatively stable due to non renormalization theorems. Specially for large $\bar\sigma > 10^{16}\ GeV$ the Type I Susy seesaw can provide a rather attractive inflationary seesaw with a natural explanation for neutrino masses and weaker tuning demands on the radiatively unstable Susy breaking parameters than the extreme and unstable fine-tunings demanded by typical inflection point scenarios and in particular the Dirac neutrino model [6]. Moreover, unlike the chaotic sneutrino inflaton scenario[21, 22], no trans-Planckian vevs are invoked.

4 Reheating and Leptogenesis

After inflation concludes the energy stored in the inflaton will be transferred into a thermal bath of the MSSM degrees of freedom. Determination of the time required to thermalize the inflaton energy and the resulting reheat temperature T_{rh} (i.e the maximum temperature of the thermal bath after thermalization) requires understanding the post-inflationary dynamics of the LHN flat direction inflaton. An important issue that can be tackled at the level of the effective SIMSSM is generation of the the cosmological baryon number asymmetry(n_B/n_γ) via Leptogenesis[10]. Although a detailed analysis of these issues requires a separate publication, the existence of previous detailed studies of preheating[11] in a MSSM flat direction inflaton model[24] and of non-thermal Leptogenesis in a preheating model[12] make the generalizations required to combine the two ideas in the context of su-

persymmetric seesaw inflation easy to outline, but too long to derive, here. Supersymmetric seesaw inflation offers an attractive synthesis fulfilling the need expressed in [12] :

"*There have been many models of leptogenesis. A hallmark of our model is the economy of fields. The only undiscovered fields are the inflaton, ϕ, the standard model Higgs, h, and the right-handed neutrino, N. There are very good reasons for suspecting that all exist! The only unfamiliar aspect of our model is the strong coupling of the inflaton field to the Higgs field. While there is no reason to preclude such a coupling, it would be very interesting to find particle-physics models with a motivation for the coupling.*"

In our model the the inflaton is itself partly comprised of the Higgs field and therefore fulfills the requirements of [12] exactly, besides bringing together a number of other related streams of thought. We remark however that the situation is made more complex by the high reheating temperature associated with the large inflaton mass. Thus both thermal and non-thermal leptogenesis may contribute to the generation of n_B/n_γ.

Due to the gauge(H,L) and third generation yukawa(H) coupled components of the inflaton the inflaton energy is likely to decay very rapidly (i.e in decay time $\tau_{dec} << H_{infl}^{-1} \sim (hM_p)/M^2$) through the so called 'instant preheating" mechanism[11, 12, 24]. In this mechanism the preheating dynamics results in a rapid decay (well within a Hubble time) of the complete inflaton vacuum energy into a radiation bath which therefore thermalizes to a temperature determined essentially by the equality between the radiation bath energy and the starting inflaton energy. This gives an estimate for the reheating temperature

$$T_{rh} \sim T_{max} \sim V_0^{1/4} \sim M/h^{1/2} \sim 10^{11} - 10^{15}\, GeV \tag{4.1}$$

The parametric dependence is identical to that found in [24], the difference in scales arises only because the inflaton mass $M \sim 10^6 - 10^{12}$ GeV in our model is much larger than the inflaton mass $m_\phi \sim 0.1 - 10$ TeV in [24] coming from soft Supersymmetry breaking.

In the preheating mechanism a class ("χ type") of degrees of freedom, whose masses($m_\chi \sim g\phi(t)$) and decay rates ($\Gamma \sim g^3\phi(t)$) are proportional to the instantaneous inflaton value $\phi(t)$, are produced non-perturbatively every time the instanton field crosses zero. This occurs since the χ modes are ultralight for a sufficiently large time interval around the zero crossing time during which adiabaticity is violated ($\dot\omega_k > \omega_k^2$: where ω_k is the oscillation frequency at wave number k). In our model the χ modes are the components of the H, L, u_L^c, u_L chiral superfields and the $W_\pm, B$ gauge superfields. In fact the χ modes can be identified simply by checking which fields become massive in the presence of background values of the three components of the inflaton ($\tilde\nu, \tilde\nu_L^c, h^0$). Then with the usual superpotential (we have suppressed generation indices)

$$W = y^u Q_L H u_L^c + y^d Q_L \overline{H} d_L^c + y^\nu LHN + y^l L\overline{H}e_L^c + ... \tag{4.2}$$

we see that y^u leads to massive u_L, u_L^c, y^ν leads to massive e_L(one combination of the three e_L), h^0, h^+, ν_L, ν_L^c; y^l leads to massive $\bar{h}^-, e_L^c$(one combination). Since $< H, N, L >$ preserve $U(1)_{em}$, the gauge couplings give masses to Z (which forms a Dirac supermultiplet with $(\nu - \tilde{h}_0)/\sqrt{2}$) and $W_\pm$ (form a pair of Dirac supermultiplets with l^-, h_+). This set

of fields are the χ type fields whose mass varies strongly with ϕ as it oscillates and whose production, when $\phi \sim 0$, and decay when $\phi >> M_W$, is the basis of 'instant preheating'. The inflaton vev leaves the down quark and gluon/gluino fields and $\bar{h}_0$, and some combinations of the l_L^-, l_L^c fields with light (MSSM type) masses. These light (ψ-type) fields will form the first step in the decays of the χ field. As $< \phi >$ again increases the χ modes become very heavy and unstable and as a result decay rapidly(within a time $\tau_{dec} \sim \frac{h}{Mg^3} << m_\phi^{-1}$) to the light (mostly coloured) MSSM d.o.f. to which they are coupled (dominantly via the D-terms and gauge-yukawa terms but also via the Superpotential couplings for the third generation). As a result a significant fraction $\sim 10^{-1}$ of the inflaton condensate energy passes into the light MSSM modes with every crossing resulting in complete transfer within $\sim 10^2$ oscillation times.

$$\tau_{osc} \sim m_\phi^{-1} << H_{infln}^{-1} \sim (hM_p)\tau_{osc}/M \sim (1 - 150)\tau_{osc} \tag{4.3}$$

Once the energy is in the light modes MSSM interactions, in particular the gauge interactions, are sufficient to rapidly complete thermalization so that essentially all the inflaton energy will be thermalized within, at most, a few Hubble times after the end of inflation. Rapid decay of the inflaton oscillation amplitude leaves the light modes to thermalize the energy dumped by the inflaton into a radiation bath of all modes: which are no longer ever heavy because the inflaton has decayed. The reheating temperature is

$$T_{rh} \sim (\frac{30}{\pi^2 g_*})^{1/4} V_0^{1/4} \sim T_{max} \sim 10^{11} - 10^{15}\, GeV \tag{4.4}$$

where $g_* = 228.75$ is the effective number of MSSM degrees of freedom. The essential point is that the reheating temperature is well above that required to produce relativistic populations of gravitinos : which are unacceptable if their lifetimes are larger than the nucleosynthesis time $\tau_N \sim 1\, sec$ since their decay after nucleosynthesis would destroy the created nucleons. The straightforward and generic resolution of this gravitino problem is if the graviton masses are sufficiently large so that the gravitinos decay before nucleosynthesis[25]

$$\tau_{grav} \sim 10^5\, sec(\frac{1\, TeV}{m_{3/2}})^3 << \tau_N \sim 1sec \tag{4.5}$$

Thus we see that the viability of Supersymmetric seesaw Inflation strongly indicates that the scale of supersymmetry breaking -as indicated by the gravitino mass- should be above 50 TeV. The fact[16] that such large supersymmetry breaking scales are preferred by both the NMSGUT and the latest data indicating[26] light Higgs mass $M_h \sim 125\, GeV$ rounds off the picture nicely. Furthermore such large reheat temperatures also ensure abundant thermal production of all flavours of righthanded neutrinos after inflation. Their CP violating decays into leptons can generate the net lepton number density which drives creation of the requires n_B/n_γ by Sphaleron processing[10]. Thus the NMSGUT can not only accommodate inflation but is also compatible[27] with (thermal) Leptogenesis [10] for generating the observed baryon to entropy density $n_B/s \sim 10^{-10}$.

An interesting additional source of *non-thermal* leptogenesis is provided when one realizes[12] that, the Higgs field H, which is one of three fields making up the putative inflaton direction in field space, is itself a χ type field and is furthermore coupled to the righthanded neutrinos. Thus during the course of oscillations of the field components of the inflaton - which commence at the end of the inflation - the Higgs mass $m_h \sim g_2\phi$ fluctuates to values both below and above the righthanded neutrino masses which are essentially constant at $M_{\nu^c} \sim f\bar{\sigma}$ even in the presence of the inflaton (i.e $\tilde{\nu}, h, N$) background since $g_2 >> f, y$. Thus as the Higgs mass oscillates below and above the right handed neutrino masses one expects CP violating -therefore net lepton number producing - inter-conversion of the Higgs with righthanded Neutrinos as in [12]. If this net lepton number is not washed out by the inflaton energy dump (so that a Hubble volume contains a certain Lepton excess produced by this inter-conversion even though average energies are well above the mass of the right handed Higgs) then we may expect that a non-thermal Leptogenesis component will add to the thermal leptogenesis due to decay of the righthanded neutrino bath.

An important complication in the present case, that we have glossed over in the above account, is that the L, H and N components of the inflaton can have quite different decay rates once the gauge interactions are effective, since N is a gauge singlet. A proper analysis must track the evolution of all three fields making up the inflaton - from an initial condition(the end of inflation) where they start out equal. This makes the equation of motion and Boltzmann equation for the relevant degrees of freedom significantly more complex and this requires a separate numerical study which involves the interplay of the couplings f_A, y_{AB}, g_2. The study of this evolution and the operation of Leptogenesis in these models is now in progress.

5 Inflation and neutrino masses in the NMSGUT

Finally we consider the embedding of our generic Type I scenario in a realistic Susy SO(10) model[16, 17, 28] that has successfully fitted the known fermion mass-mixing data and can also be consistent with limits from B violation and other exotic processes[17]. We will see that neutrino flavour plays a key role in enabling inflation : the model favours an inflaton composed of third generation conjugate sneutrino, first generation left slepton (sneutrino) and $T_{3R} = 1/2$ Higgs.

The New Minimal SO(10) GUT (NMSO(10)GUT) uses Higgs fields in the $\mathbf{210}, \mathbf{126}, \overline{\mathbf{126}}$ representations of SO(10) which contain 5 SM singlets whose vevs break SO(10) down to the SM gauge group at a superheavy scale M_X. Three of these vevs, called p, ω, a come from the $\mathbf{210}$-plet and one each from the $\mathbf{126}(\sigma), \overline{\mathbf{126}}(\bar{\sigma})$. An explicit solution to symmetry breaking $SO(10) \rightarrow SIMSSM$, in terms of a simple cubic equation for a complex variable x and depending on a single parameter ratio ξ was found in the third paper in [15]. This solution preserves supersymmetry and makes no use of the soft breaking terms which constitute a negligible perturbation of the global susy symmetry breaking problem[29], in the sense that they modify the superheavy vevs $\sim M_X \sim 10^{17}$ GeV only by terms of order $M_S \sim 10^4 GeV$.

The spectra calculated[15, 16] using this analytic solution for the the MSGUT vacuum are the basis of our detailed Renormalization Group analysis of grand unification in this class of models. Questioning the received wisdom that large SO(10) representations make grand unification futile[30] we showed[16, 17, 31] that the inclusion of threshold corrections considerably ameliorates the problem of large gauge beta functions by allowing one to raise the threshold corrected unification scale close to the Planck scale and lower the gauge coupling at unification. Taken together these features imply that even with the huge beta functions characteristic of MSGUTs the problem of a Landau pole in the gauge coupling may be postponed to the Planck scale : where it becomes moot along with the structure of space time anyway. The physics of asymptotically strongly coupled gravity and gauge theories is anybody's guess (see however [32] for our speculations and simplified model for 'tamed' asymptotically strong GUTs). There are even claims that gravity is capable of ensuring the asymptotic freedom of any gauge theory[33, 34]. It is also possible that a RG fixed surface on which the gauge coupling remains weak in the UV may exist. In view of the many uncertainties we take the stand that the large beta functions of the NMSGUT are not an issue that need prohibit the study of these minimal and realistic theories.

The grand unified minimum of the potential defined by the vevs $\Omega \equiv \{p, \omega, a, \sigma, \bar{\sigma}\}$ shifts only by fractions of order 10^{-12} due to supergravity mediated soft supersymmetry breaking terms. The D terms of SO(10) are all exactly zero for these vevs. To examine the issue of an inflaton corresponding to the NLH flat direction in the SIMSSM we must demonstrate the existence of a corresponding flat direction of the full GUT potential based on light(SIMSSM) field vevs. This flat direction rolls out of the grand unified minimum that defines the MSGUT vacuum with the SIMSSM as its effective theory. The relevant fields are the GUT scale vev fields Ω and the (6) possible components $h_i, \bar{h}_i; i = 1...6$ of the light MSSM Higgs doublet pair $H, \overline{H}$ together with the chiral lepton fields $L_A, \nu_A^c, A = 1, 2, 3$. The relevant superpotential is then[15, 16]

$$
\begin{aligned}
W &= 2\sqrt{2}(h_{AB}h_1 - 2\sqrt{3}f_{AB}h_2 - g_{AB}(h_5 + i\sqrt{3}h_6)) + \bar{h}^T \mathcal{H}(< \Omega >)h \\
&\quad + 4\sqrt{2}f_{AB}\bar{\sigma}\bar{\nu}_A\bar{\nu}_B + W_\Omega(\Omega)
\end{aligned} \tag{5.1}
$$

where

$$
\begin{aligned}
W_\Omega(\Omega) &= m(p^2 + 3a^2 + 6\omega^2) + 2\lambda(a^3 + 3p\omega^2) \\
&\quad + (M + \eta(p + 3a - 6\omega))\sigma\bar{\sigma}
\end{aligned} \tag{5.2}
$$

and

$$
\frac{\partial W_\Omega}{\partial \Omega}|_{h,\bar{\nu},L=0} = 0 \qquad D_\alpha(\Omega)|_{h,\bar{\nu},L=0} = 0 \tag{5.3}
$$

here h_{AB}, g_{AB}, f_{AB} are the yukawa coupling matrices of the three matter 16-plets to the **10, 120, $\overline{\mathbf{126}}$** Higgs multiplets respectively. Equation (5.3) defines the MSGUT vacuum[15].

Of the 5 diagonal D-terms of SO(10) only those corresponding to the generators T_{3L}, $T_{3R}, B - L$ are charge and color neutral with vevs for Ω and ν, ν^c, h_0. The vevs Ω do not

contribute to these D terms so their values are

$$D_{3L} = \frac{g_u}{2}(-\sum_{i=1}^{6} |h_{i0}|^2 + \sum_A |\tilde{\nu}_A|^2) \tag{5.4}$$

$$D_{3R} = \frac{g_u}{2}(\sum_{i=1}^{6} |h_{i0}|^2 - 2|h_{40}|^2 - \sum_A |\tilde{\bar{\nu}}_A|^2) \tag{5.5}$$

$$D_{B-L} = \sqrt{\frac{3}{8}} g_u (\sum_A (|\tilde{\bar{\nu}}_A| - |\tilde{\nu}_A|^2) + 2|h_{40}|^2) \tag{5.6}$$

where we have used the fact (paper 5 in [15] and [16]) that only $h_{4\alpha} = \Phi_{\dot{2}\alpha}^{44}$ has $B - L = +2, T_{3R} = -1/2$ and thus $T_{3L} = 1/2$ while all others have $T_{3R} = 1/2$ and $B - L = 0$. Note that g_u is the SO(10) gauge coupling in the standard unitary normalization. Thus the D-flatness conditions are

$$\sum_A |\tilde{\nu}_A|^2 = \sum_i |h_{i0}^2| = \sum_A |\tilde{\bar{\nu}}_A|^2 + 2|h_{40}|^2 \tag{5.7}$$

For simplicity we assume that only one generation each of sneutrinos ν_A and conjugate sneutrinos $\bar{\nu}_B$ contributes to the inflaton flat direction; but not that they must belong to the same generation. At this point we remind the reader[15, 16] that in MSGUTs the MSSM Higgs doublet pair is defined by fine tuning $Det(\mathcal{H}) \simeq 0$ so that its lightest eigenvalue $\mu \sim M_W \sim 1$ TeV specifies the μ term in the superpotential of the SIMSSM : $W = \mu \overline{H}H + ...$. The doublet pair $H, \overline{H}$ is a linear combination[15, 31, 35] of the 6 doublet pairs of the the NMSGUT :

$$h_i = U_{ij}H_j \qquad \bar{h}_i = \overline{U}_{ij}\overline{H}_j \tag{5.8}$$

where $U, \overline{U}$ are the unitary matrices that diagonalize the doublet mass matrix $\mathcal{H}$: $\overline{U}\mathcal{H}U = Diag\{\mu, M_2^H, ..., M_6^H\}$ to positive masses. To leading approximation they can be calculated with $\mu = 0 = Det(\mathcal{H})$. The so called Higgs fractions : $\alpha_i = U_{i1}, \bar{\alpha}_i = \overline{U}_{i1}$, are crucial in determining the grand unified formulae[15, 16] for the fermion yukawa couplings that give rise to the fermion masses. To obtain the tree level yukawa couplings one makes the replacement $h_i, \bar{h}_i \rightarrow \alpha_i H, \bar{\alpha}_i \overline{H}$ in the expressions coupling the GUT Higgs doublets($h_i, \bar{h}_i$) to the matter fermions of the SIMSSM. Thus in particular the neutrino Dirac coupling is $((\tilde{h}_{AB}, \tilde{g}_{AB}, \tilde{f}_{AB}) = 2\sqrt{2}(h_{AB}, g_{AB}, f_{AB})$)

$$y_{AB}^{\nu} = \tilde{h}_{AB}\alpha_1 - 2\sqrt{3}\tilde{f}_{AB}\alpha_2 - \tilde{g}_{AB}(\alpha_5 + i\sqrt{3}\alpha_6) \tag{5.9}$$

From the $|F_{\bar{h}}|^2$ contributions to the potential it is clear that the involvement of any but the light Higgs doublet H would lead to GUT scale rather than conjugate neutrino scale masses for the inflaton. Moreover in view of the stringent upper bounds on the fermion yukawas (see eqn.(2.22)) the involvement of the lightest generation is unavoidable. Thus we take $\nu_A = \nu_1$. However if we also take $\tilde{\bar{\nu}}_A = \tilde{\bar{\nu}}_1$ we find that the tuning constraint has *at best* the

form $|y_{11}|^2 \sim 10(|y_{21}|^2 + |y_{31}|^2)$: which is very hard to satisfy with normal neutrino mass hierarchy(the case studied so far) in (N)MSGUTs . On the other hand with $\nu_A^c = \nu_3^c$ there is a possibility of satisfying the fine tuning condition. Thus our ansatz for the flat direction fields is

$$\tilde{\nu}_1 = \frac{\phi}{\sqrt{3}} \qquad h_{i0} = \frac{\alpha_i \phi}{\sqrt{3}} \qquad \tilde{\nu}_3 = \frac{\phi}{\sqrt{3}}\sqrt{1 - 2|\alpha_4|^2} \tag{5.10}$$

Notice the peculiar role of the Higgs fraction α_4 which enters the flat direction ansatz as as $\Gamma = 1 - 2|\alpha_4|^2$. As it happens the solutions we have found earlier [16] often have $|\alpha_4| \sim 0.5$. Thus it is not inconceivable that Γ can be consistently tuned to zero by varying the GUT parameters. The challenge is to do so without destroying the realistic fits to the fermion data.

By varying the fields $\Omega, \nu_A, \bar{\nu}_A$ we can now easily derive the F-term potential

$$\begin{aligned}
V_{hard} &= \left[(y^{\nu\dagger}y^\nu)_{11} + \Gamma(|\tilde{h}_{31}|^2 + 4|\tilde{g}_{31}|^2 + (y^\nu y^{\nu\dagger})_{33}) + 4|\tilde{f}_{33}|^2\Gamma^2 \right]\frac{|\phi|^4}{9} \\
&+ \frac{8}{3\sqrt{3}}\tilde{f}_{33}|y_{31}^\nu||\bar{\sigma}|\sqrt{\Gamma}\text{Cos}(\theta_\phi + \theta_{y_{31}^\nu} - \theta_{\bar{\sigma}}))|\phi|^3 + \tag{5.11} \\
&\left[\frac{|\mu|^2}{3} + \frac{16}{3}|\tilde{f}_{33}|^2|\bar{\sigma}|^2\Gamma \right]|\phi|^2 \tag{5.12}
\end{aligned}$$

We can also write down the generic Supergravity(SUGRY)-NUHM generated soft terms in terms of a common trilinear parameter A_0 but different soft mass parameters $\tilde{m}_{\tilde{f}}^2, \tilde{m}_{h_i}^2$ for the 16 plets and the different Higgs (we have dropped the constant term from $M_S W(\Omega)$ assuming it is removed by the Supergravity scenario tuning to set the GUT scale vacuum energy to zero by tuning hidden sector parameters). The differences among the SO(10) Higgs soft masses could be due to renormalization from the threshold corrected unification scale/Planck scale to the scale $M_X^0 = 10^{16.25}$ GeV at which the SIMSSM and NMSGUT are matched in our work [36].

$$\begin{aligned}
V_{soft} &= A_0 W + c.c. + \tilde{m}_{16}^2|\tilde{\Psi}|^2 + \sum_i \tilde{m}_{h_i}^2|h_i|^2 \\
&= 2A_0\sqrt{\Gamma}|y_{31}^\nu|\frac{|\phi|^3}{3\sqrt{3}}\text{Cos}(3\theta_\phi + \theta_{y_{31}^\nu}) + \frac{4}{3}A_0\tilde{f}_{33}|\bar{\sigma}|\Gamma|\phi|^2\text{Cos}(2\theta_\phi + \theta_{\bar{\sigma}}) \\
&+ (\widehat{m}_0^2 - \frac{|\mu|^2}{3})|\phi|^2 \tag{5.13}
\end{aligned}$$

where

$$\widehat{m}_0^2 = \frac{\tilde{m}_{16}^2}{3}(1 + \Gamma) + \sum_i \frac{\tilde{m}_{h_i}^2|\alpha_i|^2}{3} + \frac{|\mu|^2}{3} \tag{5.14}$$

and $\tilde{m}_{16}, \tilde{m}_{h_i}, A_0$ are all $\sim O(M_S)$ Now the extreme dominance $f_{33}|\bar{\sigma}| >> M_S$ implies that the phase θ_ϕ is fixed by minimizing just the term in V_{hard}:

$$\theta_\phi = \pi + \theta_{\bar{\sigma}} - \theta_{y_{31}^\nu} \tag{5.15}$$

We shall assume that θ_ϕ is fixed at this value. Since the inflationary dynamics is at large values of $|\phi|$ and fixed θ_ϕ we can work just with a real field ϕ. Comparing the sum of the hard and soft potentials with the generic renormalizable inflaton potential in Section **2**, we immediately obtain the parameter identifications

$$
\begin{aligned}
h &= \frac{2}{\sqrt{3}}\left[(y^{\nu\dagger}y^\nu)_{11} + \Gamma(|\tilde{h}_{31}|^2 + 4|\tilde{g}_{31}|^2 + (y^\nu y^{\nu\dagger})_{33}) + 4|\tilde{f}_{33}|^2\Gamma^2)\right]^{\frac{1}{2}} \\
A &= \frac{1}{h}(16|\tilde{f}_{33}||y_{31}^\nu||\bar{\sigma}|\sqrt{\Gamma} + 4|y_{31}^\nu|\sqrt{\Gamma}A_0 Cos(3\theta_{\bar{\sigma}} - 2\theta_{y_{31}^\nu})) \\
M^2 &= \frac{32}{3}|\tilde{f}_{33}|^2|\bar{\sigma}|^2\Gamma + \frac{8}{3}A_0\tilde{f}_{33}|\bar{\sigma}|\Gamma Cos(3\theta_{\bar{\sigma}} - 2\theta_{y_{31}^\nu}) + 2\widehat{m}_0^2
\end{aligned}
\tag{5.16}
$$

The fine tuning condition $A = 4M$ now becomes

$$
\begin{aligned}
|y_{31}^\nu|^2 &= \frac{8\Lambda_n}{9\Lambda_d - 8\Lambda_n(1+\Gamma)}\left[|y_{11}^\nu|^2 + |y_{21}^\nu|^2 + \Gamma(|\tilde{h}_{31}|^2 + 4|\tilde{g}_{31}|^2 + |y_{32}^\nu|^2 + |y_{33}^\nu|^2)\right. \\
&\quad \left. + 4|\tilde{f}_{33}|^2\Gamma^2\right]
\end{aligned}
\tag{5.17}
$$

Where

$$
\begin{aligned}
\Lambda_n &= 1 + \frac{A_0}{4M_3}Cos(3\theta_{\bar{\sigma}} - 2\theta_{y_{31}^\nu}) + \frac{3\widehat{m}_0^2}{16M_3^2\Gamma} \\
\Lambda_d &= \left(1 + \frac{A_0}{4M_3}Cos(3\theta_{\bar{\sigma}} - 2\theta_{y_{31}^\nu})\right)^2
\end{aligned}
\tag{5.18}
$$

and $M_3 = \tilde{f}_{33}|\bar{\sigma}|$ Note that in view of the ratio between the soft breaking scale and the mass of the heaviest right handed neutrino, $\Lambda_{n,d}$ are both very close to unity. Thus the fine tuning condition is essentially between hard parameters as in GUTs and in sharp contrast to MSSM inflaton models[4]:

$$
\begin{aligned}
|y_{31}^\nu|^2 &= \frac{8}{1-8\Gamma}(\Gamma(|\tilde{h}_{31}|^2 + 4|\tilde{g}_{31}|^2 + |y_{32}^\nu|^2 + |y_{33}^\nu|^2) \\
&\quad + |y_{11}^\nu|^2 + |y_{21}^\nu|^2 + 4|\tilde{f}_{33}|^2\Gamma^2)
\end{aligned}
\tag{5.19}
$$

In NMSGUT fits of the fermion data we typically find a strong hierarchy $|y_{33}| >> |y_{32}| >> |y_{31}| >> |y_{21}| > |y_{11}|$. So it is evident that one must tune

$$
\Gamma \approx 0 \quad \text{i.e} \qquad |\alpha_4| \approx \frac{1}{\sqrt{2}}
\tag{5.20}
$$

to a good accuracy. This means that the MSSM doublet H is almost exactly 50% derived from the doublet in the 210 plet ! If this condition can be achieved the remaining tuning condition is only

$$
|y_{31}^\nu|^2 = 8(|y_{11}^\nu|^2 + |y_{21}^\nu|^2)
\tag{5.21}
$$

which is easy to enforce in the NMSGUT.

However there is an additional demand coming from eqn(2.22) : $h^2/M_3 \sim (y^{\nu\dagger}y^\nu)_{11}/M_3 \sim 10^{-25}$ which is, at first glance, much harder to enforce. It is rather remarkable that our results in[17] offer a quite reasonable way out of also this predicament. The point is that [17, 37], the yukawa couplings of matter fermions to the MSSM Higgs receive large wave function corrections due to the circulation of heavy fields within loops on the lines entering the yukawa vertex. As a result the tree level yukawa couplings of the NMSGUT must be dressed before they can be matched with those in the SIMSSM :

$$Y_f = (1 + \Delta_{\bar{f}}^T) \cdot (Y_f)_{tree} \cdot (1 + \Delta_f)(1 + \Delta_{H^\pm}) \tag{5.22}$$

Due to the large number of heavy fields the dressing of the Higgs fields can be rather large ($>> 10$). We already calculated[17] the dressing for the **10**-plet component of the MSSM Higgs. However in our realistic fits we find that the other components (in particular those from the **210**) can form a significant fraction of the MSSM Higgs. Above we showed that a completely independent line of argument *requires* that the doublet H be 50% derived from the **210**-plet ! Thus the lengthy calculation of the wave function corrections for each of the six GUT doublets contributing to the MSSM doublet is necessary. Even from the partial calculation[17] one can see that the large value of the wave function dressing makes the GUT tree level matter fermion yukawa couplings (i.e $\{h_{AB}, g_{AB}, f_{AB}\}_{tree}$ and therefore all the $(y_{AB}^f)_{tree}$) required to match the SIMSSM couplings at M_X^0 much smaller than they would be without these corrections! It is important to note that this reduction in SO(10) **16**-plet yukawa coupling magnitudes allows the $d = 5$ baryon violation rates - which have always been problematically large in supersymmetric GUTs - to be reduced to acceptable levels $\Gamma_{\Delta B \neq 0}^{d=5} < 10^{-34} yr^{-1}$. The NMSGUT offers a novel and structural resolution of this longstanding problem by taking seriously the non trivial wave function renormalization of the light Higgs doublets of the MSSM by the huge number of heavy fields they are coupled to. Since it is the tree level couplings that enter the formulae for the inflaton dynamics in the full GUT it is easier to satisfy eqn.(2.22). Because of this and the relatively large value of $M \sim M_3$ it should be be possible to achieve the required fine tuning once the full wave function dressing is computed.

The embedding in the GUT has overturned our naive assumption that the lowest intermediate scale would govern inflation. Instead it is rather the largest. While setting us the problem of finding solutions to the tuning condition, compatible both with an accurate fit of fermion masses and acceptable values of inflationary power spectrum and spectral index, it emphatically shows that the soft terms have little role to play in the fine tuning which belongs rather to the GUT and intermediate scale physics only. Thus the physics of SIMSSM driven inflation is in sharp contrast to the Dirac neutrino mass MSSM driven inflation[6, 7] and makes it clear that they lie counterpoised not only as regards the nature of neutrino mass but also as regards the nature of inflation and its regulating mass scale besides their degree of naturalness. Note that the quadratic dependence of corrections to soft susy parameters on the heavy masses as opposed to the logarithmic wave function normalization of superpotential parameters makes the weaker fine tuning demands on superpotential parameters only in the SIMSSM case even more appealing.

Parameter	$Value$	$Parameter$	$Value$
χ_X	0.4458	M_{h^0}	122.99
χ_Z	0.1426	M_X	7.08×10^{17}
f_3	1.066×10^{-3}	f_1, f_2	$2.59 \times 10^{-8}, 4.405 \times 10^{-5}$
h	2.44×10^{-4}	Λ_n	0.999999
M	3.043×10^{11}	Λ_d	0.999999
Γ	4.343×10^{-5}	Δ_{tuning}	0.989
$|\overline{\sigma}|$	4.69×10^{15}	M_X	5.25×10^{17}
$A_0(M_X), m_0(M_X)$	$-5.235 \times 10^5, 1.260 \times 10^4$	$\mu, B(M_X)$	$4.316 \times 10^5, -1.128 \times 10^{11}$
$M_{\bar{H}}^2$	-1.498×10^{11}	M_H^2	-1.448×10^{11}
$|\Delta_{H_0}|, |\Delta_{\bar{H}_0}|$	$50.254, 63.930$	$|\alpha_4|$	0.707
$M_3^{\nu^c}$	4.86×10^{13}	$M_{1,2}^{\nu^c}$	$1.181 \times 10^9, 2.01 \times 10^{12}$
$|y^\nu_{31tree}|$	1.997×10^{-4}	$|y^\nu_{21tree}|, |y^\nu_{11tree}|$	$4.489 \times 10^{-5}, 1.640 \times 10^{-6}$
$Log_{10}(h^2/M)$	-18.706	V_0, ϕ_{end}	$3.579 \times 10^{52}, 2.153 \times 10^{15}$
N_{pivot}, N_{CMB}	$54.22, 4.78 \times 10^{-4}$	Δ, β	$8.82 \times 10^{-12}, 5.92 \times 10^{-6}$

Table 1: Illustrative example of relevant parameters from an accurate fit of the fermion spectrum in the NMSGUT which is compatible with inflationary scenario. All masses are in GeV. $\chi_{X,Z}$ are the accuracies of the fits to 18 known fermion mass/mixing parameters at $M_{X,Z}$.

In Table I we give an example of the relevant parameters from an accurate fit of the complete fermion spectrum in the NMSGUT : which has also been tuned to make it as compatible as possible with the inflationary scenario presented here. The complete details regarding the fit are given as Appendix I. It is apparent that the fine tuning between the yukawas proceeds as anticipated with $1 - \Gamma = 1 = \Lambda_{n,d}$. The main problem lies in the fact that $h^2/M \sim 10^{-19}$ GeV is too large by six orders of magnitude. As a result the number of e-folds N_{CMB} is much smaller than required. However as explained the formulae used seriously underestimate the Higgs wave function corrections. Moreover the search of the huge parameter space has just begun. Thus we are confident that this problem can also be overcome and a completely realistic fit compatible with inflation achieved.

Finally we remark that the single stage breaking of the simple group $Spin(10)$ to the SM gauge group will lead to the formation of monopoles with a Kibble density $n_K \sim M_X^3$ at the time of the GUT phase transition. However inflation by 50 or more e-folds occurring long after the epoch when the SO(10) monopoles are formed will dilute the monopoles to completely levels removing any monopole problem or signal.

6 Discussion

In this paper we have shown how Supersymmetric Type-I seesaw models with the typical superpotential couplings found in MSLRMs and MSGUTs allow an attractive and natural implementation of renormalizable inflection point inflation. Inflation parameters are tied to

seesaw parameter values and the required fine tuning is less severe and more stable than in the Dirac neutrino case since it is essentially independent of the supersymmetry breaking parameters and is governed by the physics of intermediate scales $\sim 10^6 - 10^{12}$GeV. In the Dirac neutrino case [6] the opposite it is true and the inflation occurs at low scales.

The post-inflationary reheating behaviour in the our model differs from the Dirac neutrino case. The mechanism of "instant preheating"[11] applied to inflection point inflation models shows that oscillation after slow roll of a Susy flat direction inflaton [24] ensures efficient transfer of all the inflaton energy into thermalized MSSM plasma within few Hubble times after the end of inflation and consequently a high reheat temperature $T_{rh} \sim 10^{11} - 10^{15}$ GeV. Thus this type of model requires a gravitino mass larger than about $50\,TeV$ to remain consistent with Nucleosynthesis. Such large Supersymmetry breaking scales are also *required* by the NMSGUT to fit all the fermion data[16]. The high reheat temperatures and the presence of the Higgs in the inflaton sit comfortably with the requirements of thermal[22] and non thermal Leptogenesis[12]. The current work therefore extends the already wide scope of the New Minimal Supersymmetric GUT from a completely realistic theory compatible with the central paradigms of Beyond Standard Model(BSM) physics and predictive of parameters crucial to the discovery of Supersymmetry. It has been shown to potentially harbour a consistent Inflationary cosmogony tied to the central paradigms of seesaw neutrino mass and Leptogenesis. The complete calculation[27] of the wavefunction corrections to the tree level relations between SIMSSM and NMSGUT yukawa couplings will permit us to confirm the viability of our scenario in the NMSGUT context.

Acknowledgments

Appendix

Parameter	$Value$	Field $[SU(3), SU(2), Y]$	$Masses$ $(Units\ of\ 10^{16} Gev)$
χ_X	0.4458	$A[1,1,4]$	1323.58
χ_Z	0.1426	$B[6,2,5/3]$	0.1912
$f_{11}/10^{-9}$	25.8969	$C[8,2,1]$	60.14, 718.45, 741.09
$f_{22}/10^{-7}$	440.5316	$D[3,2,7/3]$	62.83, 715.80, 752.16
$f_{33}/10^{-2}$	0.1066	$E[3,2,1/3]$	0.30, 46.93, 55.30
$h_{11}/10^{-6}$	$-0.8871 + 0.1599i$		55.303, 830.73, 942.57
$h_{12}/10^{-6}$	$-9.0533 + 8.7699i$	$F[1,1,2]$	12.61, 12.61
$h_{13}/10^{-5}$	$5.8257 + 0.4100i$		44.84, 664.62
$h_{22}/10^{-5}$	$-4.1998 + 19.6580i$	$G[1,1,0]$	0.048, 0.38, 0.75
$h_{23}/10^{-4}$	$8.3003 - 4.1255i$		0.755, 32.31, 32.44
$h_{33}/10^{-3}$	$-3.7417 - 1.6595i$	$h[1,2,1]$	1.176, 40.90, 62.13
$g_{12}/10^{-4}$	$-0.1179 + 0.0940i$		1120.35, 1178.92
$g_{13}/10^{-5}$	$5.9911 - 0.5095i$	$I[3,1,10/3]$	0.67
$g_{23}/10^{-4}$	$9.1454 - 9.2551i$	$J[3,1,4/3]$	0.737, 29.75, 29.75
$\lambda/10^{-2}$	$-0.2982 - 0.3350i$		86.85, 798.73
η	$-10.1628 + 3.9777i$	$K[3,1,8/3]$	100.76, 972.38
ρ	$0.4475 - 2.1204i$	$L[6,1,2/3]$	48.91, 1571.17
k	$0.0247 - 0.0765i$	$M[6,1,8/3]$	1590.77
ζ	$1.2522 + 0.4940i$	$N[6,1,4/3]$	1582.18
$\bar{\zeta}$	$0.8170 + 0.8221i$	$O[1,3,2]$	3043.86
$m/10^{16}GeV$	0.02	$P[3,3,2/3]$	21.71, 2384.12
$m_\Theta/10^{16}GeV$	$-41.889 e^{-iArg(\lambda)}$	$Q[8,3,0]$	0.559
γ	3.78	$R[8,1,0]$	0.21, 0.82
$\bar{\gamma}$	-3.5398	$S[1,3,0]$	0.9277
x	$0.9382 + 0.6473i$	$t[3,1,2/3]$	0.60, 38.05, 94.02, 181.09
Δ_X	1.52		555.93, 755.27, 15333.10
Δ_G	-7.505	$U[3,3,4/3]$	0.786
$\Delta\alpha_3(M_Z)$	-0.004	$V[1,2,3]$	0.549
$\{M^{\nu^c}/10^{12}GeV\}$	0.001181, 2.01, 48.59	$W[6,3,2/3]$	1877.78
$\{M^\nu_{II}/10^{-12}eV\}$	0.3880, 660.09, 15968.49	$X[3,2,5/3]$	0.185, 59.281, 59.281
$M_\nu(meV)$	2.148903, 7.32, 40.17	$Y[6,2,1/3]$	0.23
$\{Evals[f]\}/10^{-6}$	0.025897, 44.05, 1065.71	$Z[8,1,2]$	0.81
Soft parameters at M_X	$m_{\frac{1}{2}} = -1062.672$	$m_0 = 12603.819$	$A_0 = -5.2347 \times 10^5$
	$\mu = 4.3160 \times 10^5$	$B = -1.1281 \times 10^{11}$	$\tan\beta = 50.0000$
	$M^2_{\bar{H}} = -1.4978 \times 10^{11}$	$M^2_H = -1.4480 \times 10^{11}$	$R_{\frac{b\tau}{s\mu}} = 1.4504$
$Max(|L_{ABCD}|, |R_{ABCD}|)$	$9.6779 \times 10^{-23}\,\mathrm{GeV}^{-1}$		

Table 2: Fit : Column 1 contains values of the NMSGUT-SUGRY-NUHM parameters at M_X derived from an accurate fit to all 18 fermion data and compatible with RG constraints. Unificaton parameters and mass spectrum of superheavy and superlight fields are also given. The values of $\mu(M_X), B(M_X)$ are determined by RG evolution from M_Z to M_X of the values determined by the EWRSB conditions.

Parameter	$Target = \bar{O}_i$	$Uncert. = \delta_i$	$Achieved = O_i$	$Pull = (O_i - \bar{O}_i)/\delta_i$
$y_u/10^{-6}$	2.031523	0.776042	2.036884	0.006908
$y_c/10^{-3}$	0.990278	0.163396	0.985427	-0.029685
y_t	0.350699	0.014028	0.350375	-0.023035
$y_d/10^{-5}$	7.314770	4.264511	8.249786	0.219255
$y_s/10^{-3}$	1.385711	0.654056	1.335549	-0.076693
y_b	0.438505	0.227584	0.496225	0.253620
$y_e/10^{-4}$	1.190847	0.178627	1.182832	-0.044871
$y_\mu/10^{-2}$	2.444540	0.366681	2.408165	-0.099201
y_τ	0.519320	0.098671	0.529217	0.100302
$\sin\theta_{12}^q$	0.2210	0.001600	0.2210	-0.0066
$\sin\theta_{13}^q/10^{-4}$	29.4299	5.000000	29.5102	0.0161
$\sin\theta_{23}^q/10^{-3}$	34.6272	1.300000	34.6440	0.0129
δ^q	60.0211	14.000000	59.9431	-0.0056
$(m_{12}^2)/10^{-5}(eV)^2$	4.8973	0.519109	4.8979	0.0012
$(m_{23}^2)/10^{-3}(eV)^2$	1.5613	0.312270	1.5600	-0.0043
$\sin^2\theta_{12}^L$	0.2939	0.058780	0.2944	0.0094
$\sin^2\theta_{23}^L$	0.4618	0.138552	0.4597	-0.0151
$\sin^2\theta_{13}^L$	0.0252	0.019000	0.0225	-0.1439
$Eigenvalues(\Delta_{\bar{u}})$	0.066017	0.066029	0.066044	
$Eigenvalues(\Delta_{\bar{d}})$	0.063539	0.063551	0.063566	
$Eigenvalues(\Delta_{\bar{\nu}})$	0.073037	0.073049	0.073064	
$Eigenvalues(\Delta_{\bar{e}})$	0.080472	0.080484	0.080499	
$Eigenvalues(\Delta_Q)$	0.061610	0.061622	0.061635	
$Eigenvalues(\Delta_L)$	0.073586	0.073599	0.073611	
$\Delta_{\bar{H}}, \Delta_H$	63.930186	50.254471		
α_1	$0.6402 + 0.0000i$	$\bar{\alpha}_1$	$0.7220 - 0.0000i$	
α_2	$0.0518 + 0.0217i$	$\bar{\alpha}_2$	$0.0387 + 0.0540i$	
α_3	$-0.0405 - 0.0412i$	$\bar{\alpha}_3$	$-0.0619 - 0.0274i$	
α_4	$-0.6968 + 0.1200i$	$\bar{\alpha}_4$	$0.6213 - 0.0161i$	
α_5	$0.1061 + 0.0735i$	$\bar{\alpha}_5$	$0.0585 + 0.0173i$	
α_6	$0.1356 - 0.2204i$	$\bar{\alpha}_6$	$0.1646 - 0.2294i$	
$|\alpha_1|, |\alpha_2|$	$0.640, 0.056$	$|\bar{\alpha}_1|, |\bar{\alpha}_2|$	$0.722, 0.066$	
$|\alpha_3|, |\alpha_4|$	$0.058, 0.707$	$|\bar{\alpha}_3|, |\bar{\alpha}_4|$	$0.068, 0.622$	
$|\alpha_5|, |\alpha_6|$	$0.129, 0.259$	$|\bar{\alpha}_5|, |\bar{\alpha}_6|$	$0.061, 0.282$	

Table 3: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.4458$. Target values, at M_X of the fermion yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. The eigenvalues of the wavefunction renormalization increment matrices Δ_i for fermion lines and the factors for Higgs lines are given, assuming the external Higgs is 10-plet dominated. The Higgs fractions $\alpha_i, \bar{\alpha}_i$ which control the MSSM fermion yukawa couplings are also given. Right handed neutrino threshold effects have been ignored. We have truncated numbers for display although all calculations are done at double precision.

Parameter	$SM(M_Z)$	$m^{GUT}(M_Z)$	$m^{MSSM} = (m + \Delta m)^{GUT}(M_Z)$
$m_d/10^{-3}$	2.90000	0.75028	3.17037
$m_s/10^{-3}$	55.00000	12.14614	51.36207
m_b	2.90000	3.19379	3.05311
$m_e/10^{-3}$	0.48657	0.45539	0.46641
m_μ	0.10272	0.09267	0.09776
m_τ	1.74624	1.71722	1.71666
$m_u/10^{-3}$	1.27000	1.10756	1.28852
m_c	0.61900	0.53583	0.62338
m_t	172.50000	143.44513	171.73898

Table 4: Values of standard model fermion masses in GeV at M_Z compared with the masses obtained from values of GUT derived yukawa couplings run down from M_X^0 to M_Z both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9}(m_i^{MSSM} - m_i^{SM})^2/(m_i^{MSSM})^2} = 0.1408$.

Parameter	$Value$	Parameter	$Value$
M_1	276.93	$M_{\tilde{u}_1}$	15629.87
M_2	942.04	$M_{\tilde{u}_2}$	15625.51
M_3	662.69	$M_{\tilde{u}_3}$	76919.36
$M_{\tilde{l}_1}$	3892.50	$A_{11}^{0(l)}$	-323296.22
$M_{\tilde{l}_2}$	283.29	$A_{22}^{0(l)}$	-322948.67
$M_{\tilde{l}_3}$	65951.57	$A_{33}^{0(l)}$	-204476.65
$M_{\tilde{L}_1}$	23375.27	$A_{11}^{0(u)}$	-391038.17
$M_{\tilde{L}_2}$	23214.39	$A_{22}^{0(u)}$	-391035.62
$M_{\tilde{L}_3}$	52427.68	$A_{33}^{0(u)}$	-211710.78
$M_{\tilde{d}_1}$	3610.56	$A_{11}^{0(d)}$	-322645.16
$M_{\tilde{d}_2}$	3604.89	$A_{22}^{0(d)}$	-322642.31
$M_{\tilde{d}_3}$	134282.49	$A_{33}^{0(d)}$	-125043.45
$M_{\tilde{Q}_1}$	17575.24	$\tan\beta$	50.00
$M_{\tilde{Q}_2}$	17572.75	$\mu(M_Z)$	351033.09
$M_{\tilde{Q}_3}$	109825.82	$B(M_Z)$	2.4726×10^{10}
$M_{\bar{H}}^2$	-1.1964×10^{11}	M_H^2	-1.3584×10^{11}

Table 5: Values (GeV) in of the soft Susy parameters at M_Z (evolved from the soft SUGRY-NUHM parameters at M_X). The values of soft Susy parameters at M_Z determine the Susy threshold corrections to the fermion yukawas. The matching of run down fermion yukawas in the MSSM to the SM parameters determines soft SUGRY parameters at M_X. Note the heavier third sgeneration. The values of $\mu(M_Z)$ and the corresponding soft susy parameter $B(M_Z) = m_A^2 \sin 2\beta/2$ are determined by imposing electroweak symmetry breaking conditions. m_A is the mass of the CP odd scalar in the in the Doublet Higgs. The sign of μ is assumed positive.

Field	$Mass(GeV)$
$M_{\tilde{G}}$	662.69
$M_{\chi^{\pm}}$	$942.04, 351033.11$
$M_{\chi^{0}}$	$276.93, 942.04, 351033.10, 351033.11$
$M_{\tilde{\nu}}$	$23375.180, 23214.295, 52427.637$
$M_{\tilde{e}}$	$3892.76, 23375.33, 277.93, 23214.55, 52422.21, 65955.95$
$M_{\tilde{u}}$	$15629.83, 17575.16, 15625.45, 17572.68, 76918.45, 109826.62$
$M_{\tilde{d}}$	$3610.66, 17575.35, 3604.97, 17572.86, 109823.42, 134284.46$
M_{A}	1112118.78
$M_{H^{\pm}}$	1112118.78
$M_{H^{0}}$	1112118.78
$M_{h^{0}}$	122.98

Table 6: Spectra of supersymmetric partners calculated ignoring generation mixing effects. Inclusion of such effects changes the spectra only marginally. Due to large values of $\mu >> M_Z, M_W$ the LSP and light chargino are essentially pure Bino and Wino($\tilde{W}_{\pm}$). The light gauginos and light Higgs h^0, are accompanied by a light smuon and sometimes selectron. The rest of the sfermions have multi-TeV masses. The mini-split supersymmetry spectrum and large μ, A_0 parameters help avoid problems with Flavor Changing Neutral Currents and Charge and Color breaking/Unbounded from below(CCB/UFB) instability[17]. The sfermion masses are ordered by generation not magnituide. This is useful in identifying the spectrum calculated including generation mixing effects. Note the very light(right) smuon.

Field	$Mass(GeV)$
$M_{\tilde{G}}$	663.15
$M_{\chi^{\pm}}$	$942.22, 351025.61$
$M_{\chi^{0}}$	$276.99, 942.22, 351025.60, 351025.60$
$M_{\tilde{\nu}}$	$23214.64, 23375.50, 52426.007$
$M_{\tilde{e}}$	$249.75, 3890.86, 23214.90, 23375.64, 52420.65, 65953.06$
$M_{\tilde{u}}$	$15626.70, 15631.41, 17574.07, 17576.34, 76909.50, 109817.78$
$M_{\tilde{d}}$	$3604.73, 3615.05, 17574.26, 17576.53, 109815.13, 134273.86$
M_{A}	1112398.16
$M_{H^{\pm}}$	1112398.16
$M_{H^{0}}$	1112398.15
$M_{h^{0}}$	122.99

Table 7: Spectra of supersymmetric partners calculated including generation mixing effects. Inclusion of such effects changes the spectra only marginally. Due to large values of $\mu >> M_Z, M_W$ the LSP and light chargino are essentially pure Bino and Wino($\tilde{W}_{\pm}$). Note that the ordering of the eigenvalues in this table follows their magnitudes, comparison with the previous table is necessary to identify the sfermions

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